MULTIMEDIA UNIVERSITY

STUDENT ID NO

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# MULTIMEDIA UNIVERSITY

# FINAL EXAMINATION

**TRIMESTER 2, 2017/2018**

## ETM 4096 - DIGITAL SIGNAL PROCESSING

(TE/RE)

7 MARCH 2018
2:30 P.M- 4:30 P.M.
(2 Hours)

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**INSTRUCTION TO STUDENT**

1. This question paper consists of **10** pages (including cover page) with 4 questions and a 6-page list of formulas given in the appendix.

2. Attempt **ALL** questions. All questions carry equal marks and the distribution of the marks for each question is given.

3. Please write all your answers in the answer booklet provided.

## QUESTION 1

(a) Let $x[n] = \{2, \underset{\uparrow}{4}, -2\}$ and $h[n] = \{3, 1, \underset{\uparrow}{1}, 2\}$ where the arrows indicate the value at $n = 0$.

Find the convolution $y[n] = x[n]*h[n]$ using multiplication method.

[8 marks]

(b) The following is the impulse response of a discrete-time system. Determine whether the system is causal and/or stable. Justify your answer.

$$h[n] = \left(\frac{1}{6}\right)^n u[n]$$

[5 Marks]

(c) Consider the parallel combination of two LTI systems in Fig. Q1 below:

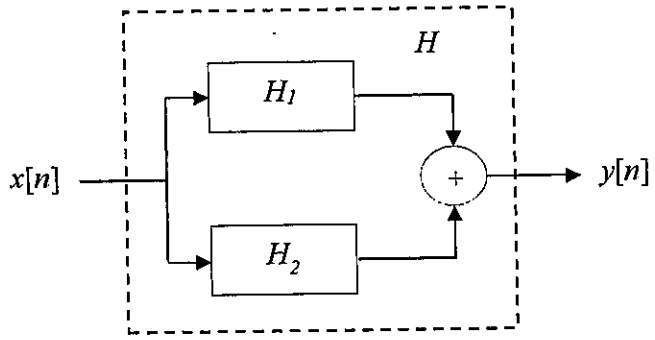



Fig. Q1

The impulse response of system $H_1$ is given by $h_1[n] = -3^n u[n]$. When the overall system input $x[n] = 2(2)^n u[n]$, the system output is observed to have the following Discrete-Time Fourier transform:

$$Y(e^{j\omega}) = \frac{4}{1 - 5e^{-j\omega} + 6e^{-2j\omega}}$$

(i) Find the impulse response $h_2[n]$ of the system $H_2$.

[8 marks]

(ii) Find the difference equation relating the input $x[n]$ and output $y[n]$ of the overall cascade system.

[4 Marks]

**Continued...**

## QUESTION 2

(a) Determine the $Z$-transform, the region of convergence (ROC) and the pole-zero plot for the following signal:

$$x[n]=\left(\frac{1}{2}\right)^n u[n]+2^n u[n]$$

[6 Marks]

(b) The signal shown in Fig. Q2 is the first cycle of a causal periodic signal that repeats periodically with a period $N_0 = 4$ for $n \geq 0$ and is zero for $n < 0$. Find the $Z$-transform of this signal given that the $Z$-transform of a causal periodic signal $x[n]$ is defined as below:

$$X(z)=\frac{X_1(z)}{1-z^{-N_0}}$$

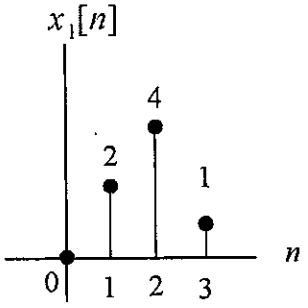



Fig. Q2

[4 Marks]

(c) A causal LTI system is characterized by the system function:

$$H(z)=\frac{2z^{-1}-0.5z^{-2}-2.5z^{-3}}{1-2.5z^{-1}+z^{-2}}$$

(i) Express the system function in the difference equation.
(ii) Determine the system impulse response, $h[n]$.
(iii) Is the system realizable? Justify your answer.

[4+10+1 Marks]

**Continued...**

## QUESTION 3

(a) Explain how the filter order affects the characteristics of the transition band of a digital filter. [4 Marks]

(b) What are the effects of using a filter with (i) linear and (ii) non-linear phase response? [4 marks]

(c) In digital filters, explain the Gibbs phenomenon and its origin. How can this problem be reduced? [4 marks]

(d) Design a lowpass filter with the following specifications:

$$0.97 < \left|H(e^{j\omega})\right| \leq 1.03 \quad \text{for} \quad |\omega| \leq 0.25\pi$$
$$\left|H(e^{j\omega})\right| \leq 0.1 \quad \text{for} \quad 0.35\pi \leq |\omega| \leq \pi$$

For the chosen window, find the impulse response function. [13 Marks]

## QUESTION 4

Given the sequence, x[n]={1,0,0,1}, find the:

a) Discrete Fourier Transform (DFT) of x[n] directly. [7 marks]

b) Inverse Discrete Fourier Transform (IDFT) of the result of part (a), to recover the original sequence x[n]. [8 marks]

c) DFT of x[n] using decimation-in-time FFT algorithm. [10 marks]

**End of page**

# APPENDIX

## SUMMARY OF USEFUL TABLES

### TABLE A1. DISCRETE-TIME FOURIER TRANSFORM PAIRS

| Sequence | Discrete-Time Fourier Transform |
|---|---|
| $\delta[n]$ | $1$ |
| $\delta[n-n_o]$ | $e^{-j\omega n_o}$ |
| $1 \ (-\infty < n < \infty)$ | $\sum_{k=-\infty}^{\infty} 2\pi\delta(\omega + 2\pi k)$ |
| $a^n u[n] \quad (\lvert a\rvert < 1)$ | $\frac{1}{1-ae^{-j\omega}}$ |
| $u[n]$ | $\frac{1}{1-e^{-j\omega}} + \sum_{k=-\infty}^{\infty} \pi\delta(\omega + 2\pi k)$ |
| $(n+1)a^n u[n] \quad (\lvert a\rvert < 1)$ | $\frac{1}{(1-ae^{-j\omega})^2}$ |
| $\frac{r^n \sin\omega_p(n+1)}{\sin\omega_p} u[n] \quad (\lvert r\rvert < 1)$ | $\frac{1}{1-2r\cos\omega_p e^{-j\omega} + r^2 e^{-j\omega})}$ |
| $\frac{\sin\omega_c n}{\pi n}$ | $X(e^{j\omega}) = \begin{cases} 1, & \lvert\omega\rvert < \omega_c \\ 0, & \omega_c < \lvert\omega\rvert < \pi \end{cases}$ |
| $x[n] = \begin{cases} 1, & 0 \le n \le M \\ 0, & otherwise \end{cases}$ | $\frac{\sin[\omega(M+1)/2)]}{\sin(\omega/2)} e^{-j\omega M/2}$ |
| $e^{j\omega_0 n}$ | $\sum_{n=-\infty}^{\omega} 2\pi\delta(\omega - \omega_0 + 2\pi k)$ |
| $\cos(\omega_0 n + \phi)$ | $\sum_{n=-\infty}^{\infty} [\pi e^{j\phi}\delta(\omega - \omega_0 + 2\pi k) + \pi e^{-j\phi}\delta(\omega + \omega_0 + 2\pi k)]$ |

## TABLE A2. SOME COMMON $z$-TRANSFORM PAIRS

### The $z$-transform

Properties of the $z$-transform

| Property | $x[n]$ | $X(z)$ | $\mathcal{R}_x$ |
|---|---|---|---|
| Linearity | $ax[n]+by[n]$ | $aX(z)+bY(z)$ | $\mathcal{R}_x \cap \mathcal{R}_y$ |
| Time shifting | $x[n-m]$ | $z^{-m}X(z)$ | $\mathcal{R}_x$ |
| Time reversal | $x[-n]$ | $X(z^{-1})$ | $1/\mathcal{R}_x$ |
| Convolution | $x[n]*y[n]$ | $X(z)Y(z)$ | $\mathcal{R}_x \cap \mathcal{R}_y$ |

Common $z$-transform pairs

| $x[n]$ | $X(z)$ | $\mathcal{R}_x$ |
|---|---|---|
| $\delta[n]$ | $1$ | $\forall z$ |
| $\delta[n-n_0]$ | $z^{-n_0}$ | Possibly $\forall z$ |
| $u[n]$ | $\frac{1}{1-z^{-1}}$ | $\lvert z\rvert > 1$ |
| $-u[-n-1]$ | $\frac{1}{1-z^{-1}}$ | $\lvert z\rvert < 1$ |
| $a^n u[n]$ | $\frac{1}{1-az^{-1}}$ | $\lvert z\rvert > \lvert a\rvert$ |
| $-a^n u[-n-1]$ | $\frac{1}{1-az^{-1}}$ | $\lvert z\rvert < \lvert a\rvert$ |
| $na^n u[n]$ | $\frac{az^{-1}}{(1-az^{-1})^2}$ | $\lvert z\rvert > \lvert a\rvert$ |
| $-na^n u[-n-1]$ | $\frac{az^{-1}}{(1-az^{-1})^2}$ | $\lvert z\rvert < \lvert a\rvert$ |

## TABLE A3. SOME COMMON WINDOWS FOR FIR FILTER DESIGN

Ideal bandpass, n=0, 1, 2 …., N (Filter order given by N)

$$h[n]=\frac{w_2}{\pi}\sin c\left(\frac{w_2\left(n-\frac{N}{2}\right)}{\pi}\right)-\frac{w_1}{\pi}\sin c\left(\frac{w_1\left(n-\frac{N}{2}\right)}{\pi}\right)$$

| | |
|---|---|
| Rectangular | $w[n]=\begin{cases}1 & 0\le n\le N\\ 0 & else\end{cases}$ |
| Hanning | $w[n]=\begin{cases}0.5-0.5\cos\left(\frac{2n\pi}{N}\right) & 0\le n\le N\\ 0 & else\end{cases}$ |
| Hamming | $w[n]=\begin{cases}0.54-0.46\cos\left(\frac{2n\pi}{N}\right) & 0\le n\le N\\ 0 & else\end{cases}$ |
| Blackman | $w[n]=\begin{cases}0.42-0.5\cos\left(\frac{2n\pi}{N}\right)+0.08\cos\left(\frac{4n\pi}{N}\right) & 0\le n\le N\\ 0 & else\end{cases}$ |
| Kaiser | $w[n]=\frac{I_0\left(\beta\sqrt{1-\left(\frac{n-\alpha}{\alpha}\right)^2}\right)}{I_0(\beta)}$ $\quad 0\le n\le N \quad \alpha=N/2$<br>where<br>$\beta=\begin{cases}0.1102(A-8.7), & A>50\\ 0.5842(A-21)^{0.4}+0.07886(A-21), & 21\le A\le 50\\ 0, & A<21\end{cases}$<br>$N=\begin{cases}(A-7.95)/(2.285\Delta\omega), & A\ge 21\\ 5.655/\Delta\omega, & A<21\end{cases}$ |

## TABLE A4. COMPARISON OF COMMONLY USED WINDOWS

| Type of Window (Filter length = $N$+1) | Peak Side-Lobe Amplitude (Relative) | Approximate Width of Main Lobe | Stopband Attenuation, $20\log_{10}\delta$ ($dB$) | Transition Width ($\Delta\omega$ $\|w_p-w_s\|$) |
|---|---|---|---|---|
| Rectangular | -13 | $4\pi/(N+1)$ | -21 | $1.8\pi/N$ |
| Bartlett | -25 | $8\pi/N$ | -25 | |
| Hanning | -31 | $8\pi/N$ | -44 | $6.2\pi/N$ |
| Hamming | -41 | $8\pi/N$ | -53 | $6.6\pi/N$ |
| Blackman | -57 | $12\pi/N$ | -74 | $11\pi/N$ |

## TABLE A5. IIR FILTER DESIGN

### TABLE A5.1. THE COEFFICIENTS IN THE SYSTEM FUNCTION OF A NORMALIZED BUTTERWORTH LOW PASS FILTER ($\Omega_c$=1) FOR ORDERS $1 \le N \le 8$

| $N$ | $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ | $a_6$ | $a_7$ | $a_8$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 1.0000 | | | | | | | |
| 2 | 1.4142 | 1.0000 | | | | | | |
| 3 | 2.0000 | 2.0000 | 1.0000 | | | | | |
| 4 | 2.6131 | 3.4142 | 2.6131 | 1.0000 | | | | |
| 5 | 3.2361 | 5.2361 | 5.2361 | 3.2361 | 1.0000 | | | |
| 6 | 3.8637 | 7.4641 | 9.1416 | 7.4641 | 3.8637 | 1.0000 | | |
| 7 | 4.4940 | 10.0978 | 14.5918 | 14.5918 | 10.0978 | 4.4940 | 1.0000 | |
| 8 | 5.1258 | 13.1371 | 21.8462 | 25.6884 | 21.8462 | 13.1372 | 5.1258 | 1.0000 |

### TABLE A5.2. CLOSED-FORM EXPRESSION FOR SOME USEFUL SERIES

$$\sum_{n=0}^{N-1} a^n = \frac{1-a^N}{1-a}$$

$$\sum_{n=0}^{N-1} na^n = \frac{(N-1)a^{N+1} - Na^N + a}{(1-a)^2}$$

$$\sum_{n=0}^{N-1} n = \frac{1}{2}N(N-1)$$

$$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a} \qquad |a|<1$$

$$\sum_{n=0}^{\infty} na^n = \frac{a}{(1-a)^2} \qquad |a|<1$$

$$\sum_{n=0}^{N-1} n^2 = \frac{1}{6}N(N-1)(2N-1)$$

$$\sum_{n=N_1}^{N_2} a^n = \frac{a^{N_1-1} - a^{N_2}}{1-a}$$

## TABLE A6. PARAMETERS OF ANALOG FILTER PROTOTYPES

Discrimination factor $d = \left[\dfrac{(1-\delta_p)^{-2}-1}{\delta_s^{-2}-1}\right]^{1/2}$

Selectivity factor $k = \dfrac{\Omega_p}{\Omega_s}$

| | |
|---|---|
| Butterworth | Order, $N \geq \dfrac{\log d}{\log k}$<br>System function, $H_a(s) = \prod_{k=0}^{N-1} \dfrac{-s_k}{s - s_k}$<br>where $s_k = \Omega_c e^{j(2k+1+N)\pi/(2N)}$ |
| Chebyshev-I | Order, $N \geq \dfrac{\cosh^{-1}(1/d)}{\cosh^{-1}(1/k)}$<br>System function, $H_a(s) = H_a(j0) \prod_{k=0}^{N-1} \dfrac{-s_k}{s - s_k}$<br>where<br>$s_k = -\Omega_p \sinh[\sinh^{-1}(1/\epsilon)/N] \sin[(2k+1)\pi/(2N)]$<br>$+ j\Omega_p \cosh[\sinh^{-1}(1/\epsilon)/N] \cos[(2k+1)\pi/(2N)]$<br>$\epsilon = \sqrt{(1-\delta_p)^{-2}-1}$<br>$H_a(j0) = \begin{cases} 1, & \text{odd } N \\ (1+\epsilon^2)^{-0.5}, & \text{even } N \end{cases}$ |
| Chebyshev-II | Order, $N \geq \dfrac{\cosh^{-1}(1/d)}{\cosh^{-1}(1/k)}$<br>System function, $H_a(s) = \prod_{k=0}^{N-1} \dfrac{s_k}{s'_k}\left(\dfrac{s - s'_k}{s - s_k}\right)$<br>where<br>$s_k = \Omega_s\{-\sinh[\sinh^{-1}(1/\epsilon)/N] \sin[(2k+1)\pi/(2N)]$<br>$+ j\cosh[\sinh^{-1}(1/\epsilon)/N] \cos[(2k+1)\pi/(2N)]\}^{-1}$<br>$s'_k = j\Omega_s / \cos((2k+1)\pi/(2N))$<br>$\epsilon = 1/\sqrt{\delta_s^{-2}-1}$ |

## TABLE A7. DISCRETE-TIME FOURIER ANALYSIS

The discrete Fourier transform (DFT)

$$\text{DFT:}\quad X(k) = \sum_{n=0}^{N-1} x(n)e^{-j2\pi kn/N}, \qquad k = 0, 1, 2, \ldots, N-1$$

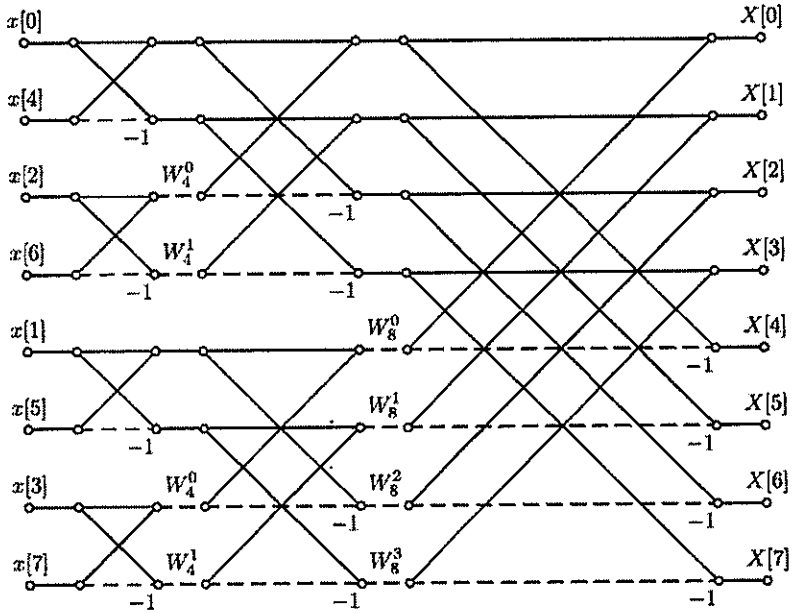
The inverse discrete Fourier transform (IDFT)

$$\text{IDFT:}\quad x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)e^{j2\pi kn/N}, \qquad n = 0, 1, 2, \ldots, N-1$$

**Properties of the DFT**

| Property | $x[n]$ | $X[k]$ |
|---|---|---|
| Linearity | $ax[n] + by[n]$ | $aX[k] + bY[k]$ |
| Time shifting | $x[\langle n - n_0\rangle_N]$ | $X[k]W_N^{kn_0}$ |
| Frequency shifting | $x[n]W_N^{-k_0 n}$ | $X[\langle k - k_0\rangle_N]$ |
| Time reversal | $x[\langle -n\rangle_N]$ | $X[\langle -k\rangle_N]$ |
| Conjugation | $x^*[n]$ | $X^*[\langle -k\rangle_N]$ |
| Convolution | $x[n] \circledast y[n]$ | $X[k]Y[k]$ |
| Modulation | $Nx[n]y[n]$ | $X[k] \circledast Y[k]$ |

**The Decimation-in-Time FFT**

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